Moduli-Constrained Phase Collapse: Embedding Quantum Collapse Geometry in Twistor Cohomology

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Abstract

We develop a formal embedding of Quantum Collapse Geometry (QCG) into the framework of twistor theory. By constructing a correspondence between the phase lattice structure of QCG and the moduli space of twistor congruences, we define a Lagrangian over the twistor moduli space whose critical points yield the collapse lattice in spacetime. We then model collapse events as singular cohomology classes of holomorphic sheaves over projective twistor space $\mathbb{CP}^3$, establishing a precise correspondence between phase coherence breakdown and global topological obstructions. This work provides a new foundation for interpreting collapse as a geometric and cohomological phenomenon, paving the way for further exploration into algebraic structures underlying quantum emergence.

1 Introduction

Quantum Collapse Geometry (QCG) proposes that spacetime and physical structure emerge from phase-resolved collapse events occurring across a probabilistic field. In this framework, collapse is not a mysterious projection but a physical, symmetry-locking process that stabilizes coherent structure. Twistor theory, pioneered by Penrose, similarly proposes that spacetime is emergent, arising from the intersection of complex null geodesics in projective twistor space. In this paper, we unify these two perspectives by embedding QCG's collapse structure into the moduli space of twistor congruences.

2 Phase Lattice in Quantum Collapse Geometry

The QCG framework models spacetime as the projection of a collapse-regulated phase field $\phi(x)$, where collapse occurs at nodes defined by:

$$\Phi[\psi] = |\nabla\phi(x)|^2 - \left|\left\langle \hat{O} \right\rangle\right|^2 \geq \epsilon \tag{1}$$

Each node in the phase lattice corresponds to a collapse event — a topological fixpoint where phase coherence crosses a threshold.

3 Twistor Congruences and Moduli Space M

A twistor $Z^\alpha = (\omega^A, \pi_{A'})$ encodes a null geodesic in spacetime. The space of all congruences of such null rays forms a moduli space $\mathcal{M}$. Each point in spacetime corresponds to a sphere of twistors; conversely, a congruence that focalizes at a real point corresponds to a collapse node in QCG.

4 Collapse Lagrangian over $\mathcal{M}$

We define a functional over the moduli space:

$$\mathcal{L}[\mathcal{C}] = \int_{\mathcal{C}} \left(\left|\bar{\partial}\Psi(Z)\right|^2 + \gamma |\nabla_{\mathcal{C}}\Psi(Z)|^2 + V(\Psi(Z)) \right) d\mu \tag{2}$$

Critical points of $\mathcal{L}$ correspond to coherent congruences that define collapse nodes:

$$\Lambda_{\text{QCG}} = \left\{ x \in \mathbb{R}^{1,3} \mid \delta\mathcal{L}[\mathcal{C}_x] = 0 \right\} \tag{3}$$

5 Sheaf Cohomology and Collapse Singularities

Let $\mathcal{F}$ be a sheaf of holomorphic sections $\Psi(Z) \in \mathcal{O}(n)$. Collapse occurs at points where local coherence fails to glue globally:

$$[\delta\Psi] \in H^1(\mathbb{CP}^3, \mathcal{F}) \neq 0 \tag{4}$$

This singular cohomology class corresponds, via the Penrose transform, to a classical collapse node in spacetime.

6 Interference and Entanglement in Sheaf Overlaps

We now extend the framework to model entanglement as a phenomenon arising from constructive interference in overlapping sheaf sections. Consider two local patches $U_i, U_j \subset \mathbb{CP}^3$ with sheaf sections $\Psi_i, \Psi_j \in \Gamma(U_i, \mathcal{F})$ and $\Gamma(U_j, \mathcal{F})$, respectively. In the overlap region $U_{ij} = U_i \cap U_j$, we define an interference functional:

$$\mathcal{I}_{ij} = \int_{U_{ij}} \left| \Psi_i(Z) - e^{i\theta_{ij}(Z)} \Psi_j(Z) \right|^2 d\mu \tag{5}$$

Here, $\theta_{ij}(Z)$ is a relative phase function capturing the phase shift between overlapping sheaf sections. If $\mathcal{I}_{ij} \to 0$, we observe maximal constructive interference — interpreted as strong entanglement.

We incorporate entropy into the model by modulating the interference with entropy weights derived from line bundle degree:

$$\tilde{\mathcal{I}}_{ij} = \mathcal{I}_{ij} \cdot \left(\frac{1}{1 + S_i(n)} + \frac{1}{1 + S_j(n)} \right) \tag{6}$$

Finally, we define an entanglement index:

$$\mathcal{E}_{ij} = \exp\left(-\tilde{\mathcal{I}}_{ij}\right) \tag{7}$$

This index ranges from 0 (fully decoherent) to 1 (perfect entanglement), and allows us to model the persistence of quantum correlations in the presence of local entropy gradients. This framework unifies phase coherence, entropy topology, and quantum entanglement in a geometric formalism.

7 Entropy Collapse as Sheaf Extension Obstructions

We now reinterpret the entropy-collapse functional $\Phi[\psi]$ in terms of obstruction theory in sheaf cohomology. Given two coherent sheaves $\mathcal{F}$ and $\mathcal{G}$, the group $\mathrm{Ext}^1(\mathcal{F}, \mathcal{G})$ measures the class of nontrivial extensions:

$$0 \to \mathcal{G} \to \mathcal{E} \to \mathcal{F} \to 0 \tag{8}$$

In this context, $\Phi[\psi]$ quantifies the failure of global coherence across overlapping regions, corresponding to the norm of the extension class $[\mathcal{E}] \in \mathrm{Ext}^1(\mathcal{F}, \mathcal{G})$. Thus,

$$\Phi[\psi] > \epsilon \quad \Leftrightarrow \quad [\mathcal{E}] \neq 0 \in \mathrm{Ext}^1(\mathcal{F}, \mathcal{G}) \tag{9}$$

Furthermore, via the Čech cohomology correspondence,

$$H^1(\mathbb{CP}^3, \mathcal{F}) \cong \check{H}^1(\{U_i\}, \mathcal{F}) \cong \mathrm{Ext}^1(\mathbb{1}, \mathcal{F}) \tag{10}$$

Collapse occurs precisely when the sheaf cohomology class is nonzero, and $\Phi[\psi]$ reflects the energy of that topological obstruction.

8 Conclusion and Outlook

This work formally unifies QCG with twistor geometry by embedding collapse events into cohomological structures over projective twistor space. In future work, we aim to compute a toy model collapse index over a patch of $\mathbb{CP}^3$, explore interference patterns in sheaf overlaps to model entanglement, and relate the entropy collapse condition $\Phi[\psi]$ to Ext groups and Čech cohomology.

References

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